Maximum and minimum values, cosmology, a conjecture and Beckenstein bound

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Abstract

A list of maximal and minimal values of physical quantities is presented. A conjecture states that the ratios (”max / min”) of maximal and minimal theoretical and often unattainable values (large numbers) for physical quantities are always powers of a constant f. It stresses that length, time, velocity, electric intensity and temperature have the same max / min ratio (span) that we call f. Energy, mass, entropy, action and momentum have another max / min ratio, which is the square of f. acceleration, charge density, current density have exponent 3, force, power, inertia momentum, electric and magnetic fields have exponent 4, pressure, density, potential vector have exponent 5, and magnetic flux has exponent 6. It is also shown that Planck quantities are sometimes the geometric means of ”max / min”. Use of f is provided in cosmology, which illustrates ”physics with constants”. Funkhouser’s large numbers are explained. This study uses quantum mechanics, cosmology, thermodynamics and special and general relativity. It is also shown that all values are derived from three pairs of values, for length (time), mass and force. Decomposition of the large numbers show that a factor $\frac{c^2}{4G}$ is contained in all formulas for length, mass and force, except for maximum length. This factor is interpreted as a general relativity effect acting on those values. It shows in particular that minimum length (Planck length) is the product of mc^2, the energy of any mass, by the Compton length of this mass, divided by $\frac{c^4}{4G}$ (purely general relativistic, Schiller). This shows that Planck length is by nature a combination of relativistic quantum mechanics and general relativistic. A similar decomposition of the maximum entropy shows that this value should be central in quantum cosmology. A calculation using quantum loop gravity shows that the sum of the surfaces of quanta of space is f times the surface of the Universe (horizon). Therefore the holographic principle is confirmed,

with an additional information. A physics without infinities is finaly suggested.

1. Introduction

The story of large numbers starts with (Dirac 1937), (Dirac 1938), (Dirac 1974) and (Eddington 1931) and continue with various authors (Hajdukovic 2012), (Alonso-Faus 2008), etc.). The present research has been inspired at its beginning, 33 years ago, by Dirac's Large Number Hypothesis, but it has no relationship to this hypothesis anymore, as no mass of particles is studied here, and the values for the large numbers are different and physical constants do not vary with time, except for f.

We define the max / min ratio of a physical quantity as the dimensionless number which is the ratio of the largest possible value for this quantity over the smallest non zero possible value for this quantity.

The max / min ratio for length and time is:

$$f = \frac{l_{\mathrm{Universe}}}{l_{\mathrm{Planck}}} = \frac{t_{\mathrm{Universe}}}{t_{\mathrm{Planck}}} = \frac{c^{5/2}}{H_0\sqrt{2}G^{1/2}\hbar^{1/2}}.$$

The actual approximate numerical value of f is $5.67\ 10^{60}$ (pure number).

f is the base for all other max / min. I have defined H_0 as equal to the inverse of elapsed time since Big Bang and

$$l_{\mathrm{Universe}} = \frac{c}{H_0} = \text{characteristic length of the Universe} = \text{Hubble radius}.$$

As f is the ratio of the time elapsed since Big Bang, estimated at 13.7 billion years, over the minimum time which has been derived as Planck time, $5.38\ 10^{-44}s$, f is not a constant, it is proportional to the time elapsed since Big Bang.

Maximum and minimum values for various physical quantities have been derived by several authors, which we shall cite in section 2, where we derive a few values as examples. We show the basis of the calculations of these quantities in tables 1 and 2. Some calculations come from physical laws, some other from physical considerations, where a maximal (minimal) value is often a product of other maximal (minimal) values. It is that sort of "logical" values that would give the quantity S that we have put in table 2.

We also present in table 2 the exponent n of f for each one and the numerical values. This is the main result for this list of values. Many values have already found by Christoph Schiller in his internet book "Mountain" (Schiller 2012). Schiller didn't stress the ratios, only some quantities.
We put in table 2 the quantity s which is the sum of the absolute values of exponents of $MKS - Cb$ units (except electromagnetism where we put [Coulomb] = 0 like in cgs). The reason for that is that, due to the technique of calculation of maxima and minima, s is often equal to n, except when a physical law alters the formula.
We also put the exponent of mass in Planck units. It shows that n is not linked to Planck units.
Finally, we put the "dimension", which is defined by the basic operations made in the calculations of max / min. We can see that the base for "dimension" is composed by length L, mass M and force F: all calculations are eventually made out of these three units, even in electromagnetism (e.g. magnetic field max/min is defined by force, electric current max/min by length, and electric potential max/min by mass).

2. Calculation of some values

The calculations hereafter often use well-known physical laws as maximum speed (Einstein), minimum action (quantum mechanics), maximum force and power (general relativity, Schiller), minimum action and spin (quantum mechanics), minimum mass (relativistic quantum mechanics), maximum mass (general relativity and cosmology), minimum entropy (thermodynamics) and minimum charge (electromagnetism), which makes the max/min ratio (ratio of the maximum over the minimum values) different than it would have been by mere "logical" calculation (n different from s). The values for f^n take in account calculations shown in tables 1 and 2. We present here some of the calculations.

- Length L: (Nottale 2003), (Rovelli 2004), (Girelli 2004), (Schiller 2012), (Garay 1995), (Calmet 2005), (Tomassini 2011): f

- v=speed:

$$v_{\max} = c \quad \text{(Einstein)},$$
$$v_{\min} = \frac{l_{\min}}{t_{\max}} \quad \text{(Schiller Mountain p.1366)}$$

- M=mass:

$$M_{\max} = \frac{c^3}{4GH_0}$$

(Herzenberg 2006), (Schiller 2012) p.1369 : $M_{\max} = \frac{F_{\max} L_{\max}}{c^2}$, only achieved by black holes; (Funkhouser 2006b), (Funkhouser 2006b), (Funkhouser 2006c), (Fahr 2006a), (Fahr 2006b); (Nottale 2003) p.18 and references herein - due to machian Universe in his demonstration; (Haikonen 2010): $M_U c^2 = \frac{GM_U^2}{L_{\max}}$),

Nottale derives the relation:

$$\frac{2GM_U}{c^2 R_U} = 1,$$

which is equivalent to our relation.

$M_{\min} = \frac{\hbar}{2cL_{\max}}$=minimum mass of vaccum according to uncertainty relation (Schiller p.1369, Choudhury, Böhmer: uncertainty relation gives $M_{\min} c^2 = \frac{\hbar}{\Delta t} = \hbar H$, therefore

$$M_{\min} = \frac{\hbar H_0}{2c^2}$$

(for Δt = age of the Universe), therefore

$$\frac{M_{\rm max}}{M_{\rm min}} = f^2$$

- A=action:

$$A_{\rm max} = E_{\rm max} t_{\rm max} = \frac{M_{\rm max} c^2 L_{\rm max}}{c} = \frac{c^5}{4GH_0^2}$$

(Schiller p.1366, maximum energy during the duration of the Universe),

$$A_{\rm min} = \frac{\hbar}{2} \quad \text{(Quantum mechanics)}$$

$$\text{therefore:} \quad \frac{A_{\rm max}}{A_{\rm min}} = f^2$$

- ρ=density:

$$\rho_{\rm max} = \frac{M_{\rm max}}{V_{\rm min}} = \sqrt{\frac{c^{15}}{128 G^5 \hbar^3 H_0^2}}$$

((Schiller 2012) p.1361 and 1369, (Choudhury 2001) – volumic mass of the initial cosmological singularity, cosmic particles have Planck mass, minimum size location is Planck length = hard repulsive core surrounded by gravitational force, lowest energy state for bosons at T tends towards 0− , (Arbab 2001), mass of the Universe over Planck volume, (Sahni 2000), (Alonso-Faus 2008).

$$\rho_{\rm min} = \frac{M_{\rm min}}{V_{\rm max}} = \frac{\hbar}{2cL_{\rm max}^4} = \frac{\hbar H_0^4}{2c^5}$$

((Schiller 2012) p.1369 - zero energy photon in vacuum, uncertainty relation for E/c extended to volume with radius of Universe size gives density of vacuum- , (Choudhury 2001), mass of the vacuum over volume of the Universe, (Arbab 2001), therefore

$$\frac{\rho_{\rm max}}{\rho_{\rm min}} = f^5$$

- T=temperature:

$$T_{\rm max} = \frac{\hbar v_{\rm max}}{k_B},$$

therefore:

$$T_{\max} = \sqrt{\frac{\hbar c^5}{2Gk_B^2}}$$

(Arbab 2003 – Planck mass times c^2 over k_B, reference 1264 of Schiller),

- S=entropy:

$$S_{\max} = \frac{M_{\max}c^2}{T_{\min}} = \frac{k_B c^5}{4G\hbar H_0}$$

((Schiller 2012) p.1369, thermal energy of the Universe over $T_{\min}$=derived above; (Schiller 2012) p.1124, reference 1006, derived from Beckenstein; (Alonso-Faus 2008), (Fullana 2012)

$$S_{\min} = \frac{k_B}{2}$$

(reference 1263 of (Schiller 2012)), therefore

$$\frac{S_{\max}}{S_{\min}} = f^2,$$

- Maximum force:

$$F_{\max} = \frac{c^4}{4G}$$

(Schiller)

- Minimum force

$$F_{\min} = \frac{GM^2}{L_{\max}^2} = \frac{\hbar^2 G H_0^4}{4c^6}$$

Hence f^4.

- I=electric current intensity:

$$I_{\max} = \frac{e}{t_{\min}} = e\sqrt{\frac{c^5}{2\hbar G}}$$

(Schiller 2012) p.1361, it is noticeable that Schiller took e instead of Qmax. We shall use this method for other electromagnetic quantities. Schiller (Schiller 2012) has derived other values, among which pressure, momentum, power, electric field, electric density, magnetic field, temperature. The other values have been derived according to physical laws: see tables 1 and 2.

Quantity		maximal quantity			
		formula	Calculation	Num.val.	Unit(MKSCb)
f		$c^{5/2}/H_o(2G\hbar)^{1/2}$		$6.E+60$	
fundamental quantities that one measures					
length	l	c/H_o	definition	1.E+26	m
time	t	$1/H_o$	definition	4.E+17	s
mass	m	$c^3/4GH_o$	$2M_U/c^2R_U = 1$	4.E+52	kg
electric intensity	l	$e \cdot c^5/2\hbar G)^{1/2}$	e/t_{min}	2.E+24	Cb/s
entropy	S	$k_Bc^5/4G\hbar H_0^2$	$M_{max}c^2/T_{min}$	2.E+98	$kg \cdot m^2/s^2 \cdot K$
fundamental quantities that one counts					
charge	Q	$e \cdot (c^5/2\hbar G)^{1/2}/H_0$	$I_{max}t_{max}$	9.E+41	Cb
magnetic flux	Φ	$c^5/4H_o^2Ge$	$B_{max}L_{max}^2$	1.E+106	$kg \cdot m^2/s \cdot Cb$
Intensive mechanical derived quantities					
Speed	v	c	sp.relativity	3.E+08	m/s
Acceleration	a	$(c^7/2\hbar G)^{1/2}$	v_{max}/t_{min}	4.E+51	m/s^2
Pressure	P	$(c^13/32G^3H_0^2\hbar)^{1/2}$	c^2M_{max}/L_{min}	2.E+104	$kg/m \cdot s^2$
Frequency	ω	$(c^5/2G\hbar)^{1/2}$	l/t_{min}	1.E+43	s^{-1}
density	ρ	$(c^{15}/128G^5\hbar^3H_o^2)^{1/2}$	M_{max}/V_{min}	4.E+156	kg/m^3
extensive mechanical derived quantities					
energy	E	$c^5/4GH_o$	$M_{max}c^2$	4.E+69	$kg \cdot m^2/s^2$
momentum	p	$c^4/4GH_o$	$M_{max}v_{max}$	1.E+61	$kg \cdot m/s$
action, ang.mom., spin	A	$c^5/4GH_o^2$	$E_{max}t_{max}$	2.E+87	$kg \cdot m^2/s$
Force	F	$c^4/4G$	Gnl.Relativity	3.E+43	$kg \cdot m/s^2$
Power	P	$c^5/4G$	Gnl.Relativity	9.E+51	$kg \cdot m^2/s^3$
Inertia momentum	I	$c^5/4GH_o^3$	$M_{max}R_{max}^2$	7.E+104	$kg \cdot m^2$
electromagnetic derived quantities					
electric field	E	c^4/Ge	F_{max}/e	2.E+62	$kg/s^2 \cdot Cb$
magnetic field	B	$c^3/4Ge$	F_{max}/ec	6.E+53	$kg/s \cdot Cb$
charge density	ρ_{el}	$e(c^9/8G^3\hbar^3)^{1/2}$	e/L_{min}^3	1.E+85	Cb/m^3
current density	j	$e(c^{11}/8G^3\hbar^3)$	I_{max}/L_{min}^2	4.E+93	$Cb/s \cdot m^2$
E-M potentiel vector	A_μ	$(c^9/32G^3\hbar)^{1/2}/e$	B_{max}/L_{min}	3.E+88	$kg \cdot m/s \cdot Cb$
thermodynamic derived quantity					
Temperature	T	$(\hbar c^5/2Gk_B^2)^{1/2}$	$\hbar\nu/k_B$	1.E+32	K

Table 1

Quantity		minimal quantity			n	S	Planck	
		formula	Calculation	Num.val.(MKS)				"dim"
fundamental quantities that one measures								
length	l	$(2G\hbar/c^3)^{1/2}$	definition	2.E-35	1	1	-1	L
time	t	$(2G\hbar/c^5)^{1/2}$	definition	8.E-44	1	1	-1	L
mass	m	$\hbar H_o/2c^2$	$\hbar/2cL_{max}$	1.E-69	2	1	1	M
electric intensity	l	eH_o	e/t_{max}	4.E-37	1	1	1	L
entropy	S	$k_B/2$	Thermodyn.	7.E-24	2	6	0	M
fundamental quantities that one counts								
charge	Q	e	e	2.E-19	1	0	0	L
magnetic flux	Φ	$2\hbar^3 G^2 H_o^4/ec^{10}$	$B_{min}L_{min}^2$	3.E-259	6	4	0	FL^2
Intensive mechanical derived quantities								
Speed	v	$2G\hbar H_p^2/c^3)^{1/2}$	L_{min}/t_{max}	5.E-53	1	2	0	L
Acceleration	a	$2G\hbar H_o^3/c^4$	v_{min}/t_{max}	2.E-131	3	3	1	L^3
Pressure	P	$G\hbar^2 H_o^4/c^6$	$c^2 M_{min}^2/L_{max}^4$	3.E-200	5	4	4	ML^3
Frequency	ν	H_o	l/t_{max}	2.E-18	1	1	1	L
density	ρ	$\hbar H_o^4/2c^5$	M_{min}/V_{max}	1.E-147	5	4	4	ML^3
extensive mechanical derived quantities								
energy	E	$\hbar H_o/2$	$M_{min}c^2$	2.E-52	2	5	1	M
momentum	p	$\hbar H_o/2c$	h/L_{max}	4.E-61	2	3	1	M
action, ang.mom., spin	A	$\hbar/2$	Quant.Mech.	5.E-35	2	4	0	M
Force	F	$\hbar^2 G H_o^4/c^6$	GM^2/L_{max}^2	3.E-200	4	4	2	F
Power	P	$\hbar^2 G H_o^4/c^5$	GcM_{min}^2/L_{max}^2	9.E-192	4	6	2	F
Inertia momentum	I	$\hbar^2 H_o G/c^5$	$M_{min}R_{min}^2$	7.E-139	4	3	-1	ML^2
electromagnetic derived quantities								
electric field	E	$G\hbar^2 H_o^4/ec^6$	$M_{min}/a_{min}/e$	2.E-181	4	3	2	F
magnetic field	B	$\hbar^2 G H_o^4/ec^7$	F_{min}/ec	6.E-190	4	2	3	F
charge density	ρ_{el}	eH_o^3/c^3	e/L_{max}^3	7.E-98	3	3	3	L^3
current density	j	eH_o^3/c^2	I_{min}/L_{max}^2	2.E-89	3	3	-3	L^3
E-M potentiel vector	A_μ	$2\hbar^2 G H_o^5/ec^8$	B_{min}/L_{max}	9.E-216	5	3	1	L^3
thermodynamic derived quantity								
Temperature	T	$\hbar H_o/k_B$	E_{min}/k_B	2.E-29	1	1	1	L

Table 2

3. Other related large ratios and large numbers in the literature

Nottale (Nottale 2003) has introduced a number slightly different from f, $K(=5.3\ 10^{60})$, defined as the ratio $\frac{\Lambda^{-1/2}}{l_P}$, Λ being the cosmological constant. He sets $L = \Lambda^{-1/2}$ as the unattainable maximum length. In that way, it differs from our $L_{\max} = l_{\mathrm{Universe}}$, as $L_{\max}$ is the actual maximum length, but which evolves with time, being equal to $\frac{c}{H_0}$. Nottale suggests that the cosmological constant should replace $\frac{c}{H_0}$, therefore giving a different constant f' that would be constant in time.

Since many years, some authors did found maximal or minimal possible values that physical quantities can have, or that can be measured. They invoke for this general or specific considerations based on quantum mechanics, cosmology, gravitation thermodynamics and special or general relativity. Typical examples are Planck lengths and times that are supposed to be the smallest size and duration ((Girelli 2004), (Rovelli 2004), (Garay 1995), (Tomassini 2011), (Calmet 2005), (Ray 2007).

Additional considerations by Schiller or other authors show that when a maximal value is postulated for a physical quantity, it is generally possible to derive the minimal one that can be measured. As such a value cannot be zero according (generally) to quantum considerations, the ratio of the maximal over the minimal values associated to this physical quantity is a very large but finite number.

Big numbers of the order of 10^{120} (sometimes in reference to our f^2) have been often cited recently ((Berman 2006), (Framton 2007), (Alonso-Faus 2008), (Funkhouser 2006c), (Egan 2009), (Sabbata 1994), (Ray 2007). Our value for f has been cited by Ray and al. and (Alonso-Faus 2008), for lengths. (Sahni 2000) and (Funkhouser 2006c) defined a number of the order of 10^{120} using the cosmological constant instead of Hubble constant, and therefore didn't get our formula.

Scott Funkhouser links the scaling law for the cosmological constant proposed by Zel'dovitch with numbers of the order of 10^{122}. He gives also a maximal value for entropy which he derives as $\frac{c^3}{Gh(\Lambda)} = 10^{122}$ ((Funkhouser 2006c). (Canuto 1977) and (Canuto 1978) and al. also derive a scale covariant theory of gravitation and links it with Dirac Large Number Hypothesis. (Schiller 2005) derives a minimum length, a minimum dipole moment and relates it to the lack of space-time continuity (granularity). Egan derives the maximum entropy of the Universe and finds 10^{122}. Ray states that the Planck volumic mass over the present Universe volumic mass as 10^{120}. (Berman 2006) derives the maximum angular momentum over its minimum as 10^{120} for a machian Universe. (Arbab 2003) gives a list of minimum and maximum values. (Davies 2004) derives maximum entropy as 10^{120}.

We get for that value f^2=1.34 10^{121}, always slightly different than all values cited in this paragraph.

4. Geometric means

The geometrical means for several quantities link max/min with Planck values:

$$\frac{M_{\max}}{M_p} = \frac{M_p}{M_{\min}} = f$$

Similarely, the Planck momentum p_P is the geometric mean of $p_{\max}$ and $p_{\min}$:

$$\frac{p_{\max}}{p_P} = f = \frac{p_P}{p_{\min}}$$

One has also the Planck energy E_P which is the geometric mean of $E_{\max}$ and $E_{\min}$:

$$\frac{E_{\max}}{E_P} = f = \frac{E_P}{E_{\min}}$$

(Herzenberg 2006) has computed the inertia momentum that indicates the limit between quantum mechanics and classical mechanics for free objects, and has found: $I_{\rm QM} = \frac{\hbar}{2H_0}$, which is the geometric mean between $I_{\max}$ and $I_{\min}$:

$$\frac{I_{\max}}{I_{\rm QM}} = \frac{I_{\rm QM}}{I_{\min}} = f^2$$

It is noticeable that we get the same relation for entropy as for action, which can be related to the fact that these quantities are conceptually linked according to several authors among which ((Petrova 2010).

5. f in Cosmology

In Friedman-Robertson-Walker metric for closed Universe, one has the differential equation (Atkatz 1994):

$$\dot{a}^2 + (1 + \frac{\Lambda \cdot a^2}{3}) = 0$$

for which one has the solution:

$$a(t) = c\sqrt{\frac{\Lambda}{3}} \cosh\left(t\sqrt{\frac{\Lambda}{3}}\right).$$

with :

$$\Lambda = 8\pi G \rho_{\rm vacuum}.$$

Inserting for the density of vacuum our value of minimum density gives :

$$\Lambda = \frac{8\pi G\hbar H_0^4}{c^5}.$$

And, inserting in $a(t)$, for $t = t_{\rm Planck}$:

$$a(t_P) = l_P \cdot f \cdot \cosh\left(\frac{\pi}{3f^2}\right).$$

One has $a(t_P)$ that is a function of our universal value f, which was possible by inserting all constants in the calculation, instead of setting $c = h = G = 1$. This derivation has no real physical meaning, because we used a cosmology result for a quantum cosmology situation (because of Planck time) but it is an illustration of "physics with constants".

6. f in Quantum loop gravity (QLG) and holography

We present here some results on surface, entropy and mass, using Beckenstein bound.

1) **Surfaces** QLG states (Rovelli 2004) that there are quanta of space, which are cells of about the Planck length. Their volume is roughly l_p^3, and their surface roughly l_p^2, and there are about f^3 cells, therefore about f^3 surfaces, therefore their total surface is roughly:

$$f^3 l_p^2.$$

As the surface of the Universe (if one computes its surface as the surface of S^2) is roughly

$$l_U^2,$$

the ratio (total surface of cells/surface of Universe) $= f$

The same calculation can be done with quantum formula for the surface:

$$A = \frac{8\pi\hbar G\sqrt{j_l(j_l+1)}}{c^3}$$

(Rovelli 2004) p.263. for $j = 1/2$, one has:

$$\frac{f^3 A}{4\pi L_U^2} = f,$$

same result. Hence the sum of surface of all quanta of space is f times greater than the surface of the Universe (horizon). With the Beckenstein-Hawking formula, one concludes that the Beckenstein bound of the sum of the cells is f times greater than for the Universe. A direct calculation with the entropies gives the same result:

2) **Entropy** Because of the Beckenstein-Hawking formula, this last result should be valid for entropy. The Beckenstein-Hawking formula is

$$S = \frac{k_B c^3 A}{4\hbar G}$$

If one takes the surface of the Universe as $4\pi R^2$ as for S^2:

$$S_{\mathrm{BHU}} = k_B \pi f$$

If one takes as the minimum size in Universe the aforementioned cell which has as surface operator in LQG.

$$A = \frac{8\pi\hbar G\sqrt{j_l(j_l+1)}}{c^3} \quad \text{(Rovelli p.263)}$$

and one takes the minimum quantum which is $j_l = 1/2$, one gets for the entropy of a black hole with a size of a minimum cell:

$$S_{\mathrm{BHCell}} = \pi k_B$$

Therefore, one gets: the ratio of a quantum cell and of the Universe is:

$$\frac{S_{\mathrm{BHU}}}{S_{\mathrm{BHCell}}} = f^2$$

Therefore the ratio of the entropy of ALL cells is $\frac{f^3}{f^2} = f$: The entropy contained inside all cells is f times the entropy contained inside the radius (horizon) of the Universe.

3) **Mass**

Taking the mass of a black hole to be

$$M_{\mathrm{BH}} = \sqrt{\frac{A}{16\pi\hbar^2}} \quad \text{(Eddington 1931)(Rovelli p.302)}$$

and the surface operator of a cell

$$A = \frac{8\pi\hbar G\sqrt{j_l(j_l+1)}}{c^3} \quad \text{(Rovelli p.263)}$$

with $j_l = 1/2$, one gets the mass of a minimum black hole:

$$M_{\rm BHCell} = \sqrt{\frac{G}{4\hbar c^3}}$$

Taking again the surface of the Universe as $4\pi R^2$ as for S^2, one has for a black hole of the dimension of the Universe:

$$M_{\rm BHU} = \frac{c}{2H_0\hbar}.$$

Their quotient is:

$$\frac{M_{\rm BHU}}{M_{\rm BHCell}} = f,$$

which is not the value we get for Mmax/Mmin, for evident reasons: the mass of a black hole of the size of the Universe is very different from the total mass of the Universe, and the mass of one cell of vacuum is very different as the mass of a black hole of the size of a cell. Comparing with our description of the Planck mass as geometrical mean of maximum and minimum masses, one gets 5 mass values, related by f in a intertwined way: Starting from the minimum mass, we have then Planck mass, then $M_{\rm BHCell}$, the minimum black hole mass, then maximum mass, then $M_{\rm BHU}$, the mass of a black hole the size of the Universe.

7. Discussion and conclusion

We have presented a conjecture that gives a universal value f as a base to all calculations of maximum over minimum values of physical quantities.

We claim to have explained here Funkhouser's (Funkhouser 2006c) n_1, n_2, and n_3 (to a factor π, explained here in the loop quantum gravity section, in the entropy paragraph). Let us recall that n_1 is the ratio of the Planck density to the vacuum density, for which we find f^4, n_2 is the ratio of the largest mass over the smallest mass, which is f^2, n_3 is the maximum number of degrees of freedom, which is f^2

. Funkhouser (?) wrote another paper on a hierarchy of masses in a fractal Universe. Thanks to our values, one can derive exactly the formulas presented in that paper. Unfortunately, Funkhouser made very approximate calculations, which therefore don't give realistic results.

Our quest started with two assumptions, the existence of a minimum length, assumed and sometimes demonstrated in many theories of quantum gravity, and of a maximum length, corresponding to the radius of the Universe in the case of closed Universes, to the horizon in the flat and open cases with horizon.

We gathered minimal and maximal values for the other physical quantities, from fundamental laws of from other searchers work, and derived the ones which hadn't been derived yet. Doing so, we always used a combination of the three quantities which are central in this quest: Length (or time), Mass and Force.

Therefore it is of great importance to discuss the origins and relationships of the minimal and maximal values for these three quantities.// First of all, let us see what are the origins of these six formulas:

Minimal length is derived in many ways, for different reasons cited above, and is linked with granularity. We shall see its decomposition later on, but its square contains $\frac{c^2}{4G}$

Maximal length is assumed in many theories, but remains an assumption (cosmology).

Minimum mass is demonstrated by relativistic quantum mechanics (and even by only quantum mechanics) arguments linked with the existence of quantum fluctuations and therefore could be linked to the minimum curvature of the Universe (Santos 2011). It is the product of the square of Planck length (minimum length, quantum gravity)and $\frac{c^2}{4G}$, divided by the maximum length (cosmology). We shall analyse the dependences in maximum length later on.

Maximum mass (total cosmic mass contained in the spacelike sphere which radius is the light horizon maximum length) is demonstrated in various ways, the main ones being a simple calculation from Friedmann density, and the Mach-Sciama principle. It is the product of $\frac{c^2}{4G}$ by the maximum length (cosmology).

Minimum force is derived by Newton force for minimum mass from maximum length. It is the minimum action to the power 2 (quantum mechanics) divided by c2/G (gravitation), divided by the maximum length to the power four (cosmology).

Maximum force has been demonstrated by C.Schiller and is equivalent to general relativity Einstein equation. It is the product of $\frac{c^2}{4G}$ by c square (special relativity).

A similar decomposition is possible for all maximum and minimum values, they all contain at least one of the six preceding values and they are presented in the column Dimension in the table.

There is a striking property for five of these six values (maximum length excepted): Every formula contains the term $\frac{c^2}{4G}$, which is of general relativistic origin (the maximum force is $\frac{c^4}{4G}$, and is the ratio of maximum mass over maximum length).

Its value is about 10^{28}.

f square can be decomposed in a similar way: it is the product of $\frac{c^4}{4G}$ (general relativity), the maximum length to the square (cosmology), divided by $\hbar/2$ (quantum mechanics) and c (special relativity). It therefore contains elements of each of the main theories in theoretical physics. its form (a product divided by a product) shows the large scale divided by the small scale.

f could then be the term specific for the theory unifying all fields of physics (it doesn't contain k_B, for a value containing k_B also, see later on the maximum entropy).

Let us consider the dependence in the maximum length.

The maximum mass is linear in maximum length, and increases with time, as previously noted by many authors among which Fahr. It should be due to production of matter due to quantum effects.

There is another decomposition for mass and force: $\frac{M_{max}}{M_{min}}$ is naturaly decomposable in

$\frac{L_{max}}{L_{min}}^2$. In the same way, $\frac{F_{max}}{F_{min}}$ is equal to the square of $\frac{M_{max}}{M_{min}}$.

We shall decompose now the minimum length, which is the Planck length l_p. It can be written as:

$l_p = \frac{4G}{c^4}.\hbar c/2 = \lambda c.mc^2/F_max$, where m is any mass, and λ_c is the Compton length of this mass.

We can therefore see in this decomposition that l_p is produced directly by two contributions, general relativity (Fmax) and relativistic quantum mechanics (hc/2).

The Mach-Sciama effect is instantaneous, letting think of a mechanism related to quantum mechanics, hence the expression of maximal mass within Mach-Sciama principle could lead to a new way of dealing with quantum gravity. (e.g.(Funkhouser 2003)). A complementary way at searching for quantum gravity effects is to search for maximal values containing $\hbar$, therefore quantum mechanics. It is the case for several quantities, but the best choice seems the value for maximum entropy, which contains all four usual constants, plus k_B.Its role has been outlined by others, among which Alonso-Faus (Fullana 2012). It can be decomposed in a product and quotient of k_B (thermodynamics), $\frac{c^4}{4G}$ (general relativity), $\hbar$ (quantum mechanics), H_o (cosmology), and c (special relativity). It therefore contains all theories.

We have seen in the LQG section that the Beckenstein bound for the sum of quanta of space (in the holography according to loop quantum gravity) divided by the Beckenstein bound for Universe is f: There is more information in the cells than in the Universe. This implies that all information inside the Universe is probably contained in the cells, and that even more information reside inside those cells. As our physics is not operative under this Planck length limit, on could say that more information is unavailable to us than the information available.

Holographic principle is confirmed, with an additional information on what lies outside our physics. Finally, one could ask if there is still need for infinities in physics, as the precision in any calculation and experiment never reaches the large numbers we have presented.

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